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Abstract

Operational planning within public transit companies has been extensively tackled but still remains a challenging area for operations research models and techniques. This phase of the planning process comprises vehicle scheduling, crew scheduling and rostering problems. In this paper, a new integer mathematical formulation to describe the integrated vehicle-crew-rostering problem is presented. The method proposed to solve this multi-objective problem is a sequential algorithm considered within a preemptive goal programming framework that starts from the solution of an integrated vehicle and crew scheduling problem and ends with the solution of a driver rostering problem. Feasible solutions for the vehicle and crew scheduling problem are obtained by combining a column generation scheme with a branch-and-bound method. These solutions are the input of the rostering problem, which is tackled through a mixed binary linear programming approach. An application to real data of a Portuguese bus company is reported and shows the importance of integrating the three scheduling problems.

Keywords: *binary linear programming, vehicle scheduling, crew scheduling, driver rostering, multi-objective problems.*

1. Introduction

The planning process in public transit companies includes several phases such as strategic planning, tactical planning, operational planning and real-time control (see Barnhart and Laporte, 2007). Strategic planning is concerned with network design and passenger assignment while service frequency and timetabling are defined in the tactical planning. In the operational planning, the schedule for the vehicles and the schedule for the crews must be built, for each planning period. Additionally, still in the same phase, for a given time horizon (which may include several planning periods), one must define a roster, that is, a set of lines of work, one for each particular crew, usually a driver. Finally, in the real-time control phase the whole process is evaluated, adjusted and maintained.

This paper focuses on the operational planning phase of a public transit company that operates buses to satisfy the demand of transport in an urban area.

Traditionally, this issue has been tackled on a sequential basis beginning with vehicle scheduling, followed by crew scheduling and, lastly, driver rostering. Given a set of timetabled trips, vehicle scheduling produces the schedule for the vehicles for the planning period, usually a day, thus defining vehicle blocks. Each vehicle block defines a sequence of timetabled trips to be operated by a single vehicle during a day. The crew scheduling defines the daily duties for the crews covering all vehicle blocks. Each duty is assigned to a single crew on a specific day. Finally, the crew duties that must be covered during a given time horizon are assigned to the company's drivers, thus building the roster that must comply with general legislation, labor contracts and specific institutional rules.

Due to the great complexity of the overall scheduling, it is usual to divide it into the three above combinatorial problems, which per se remain difficult, and to tackle them sequentially. However, there is a high dependency among these sub-problems. Hence, from the eighties, some authors pointed out the integration of vehicle and crew scheduling (Ball et al. (1983)) and big efforts have been made to produce efficient algorithms (Borndörfer et al. (2004), Huisman et al. (2005), Hollis et al. (2006), Mesquita and Paias (2008) and Mesquita et al. (2006)). As pointed out by these authors, this integration may lead to significant reductions in the total number of vehicles and crews. Crew-rostering integration has been devised by Caprara et al. (2001), Ernst et al. (2001), Freling et al. (2004) and Lee and Chen (2003) for other transport rostering contexts (railway and air crews) and by Chu (2007) for airport staff. In fact, the rostering problem should be integrated either with the crew scheduling problem alone or within an overall integrated scheduling process that would enable one to simultaneously solve the three problems mentioned above. To the authors' knowledge, no research has yet been published on the overall integration. However, the advantage of integrating the three problems has been pointed out in (Ernst et al. 2004) and (Borndröfer et al. 2006).

Here, the option favors the overall integration because it allows one to simultaneously analyze operating costs - vehicle plus driver costs - and other features of the final rosters, as for instance the balance of overtime work among drivers during the rostering period. Despite its computational burden, the integrated approach is expected to outperform the sequential approach in terms of solution quality.

In section 2 a review of relevant research on the integrated vehicle and crew scheduling process, as well as on the rostering issue is presented followed by a brief description of the integrated vehicle-crew-rostering problem itself (VCRP, for short). Section 3 proposes a multi-objective model for the VCRP, along with a mathematical formulation. Section 4 is devoted to the solution methodologies. The solution approach consists of applying rostering after the integrated vehicle and crew scheduling module following a preemptive goal programming approach. Some parameters controlled by the user during the optimization of the integrated vehicle and crew scheduling problem can be adjusted during the overall process in order to obtain better final solutions. In section 5, the results of a computational experiment with a set of problems taken from a real bus company are given. The conclusions, in section 6, point to the need to continue researching into solution methods to produce efficient software for a complete integration of the scheduling problems in a public transit company.

2. Relevant research for VCRP

The objective of the integrated vehicle and crew scheduling problem (VCP) is to define duties for the crews that cover at minimum cost the vehicle blocks that are obtained by optimally linking timetabled trips. The vehicle blocks are performed by a set of vehicles located at one or more depots and costs are used to report the fuel consumption of vehicles, idle times, number of vehicles, etc.

The crew duties must satisfy several labor and company rules such as maximum/minimum spread (time elapsed between the beginning and end of a crew duty), maximum/minimum effective working time, maximum/minimum break, maximum overtime work, maximum number of changeovers, etc. The cost associated to each crew duty consists usually of a fixed cost, representing the daily base-salary plus an operational cost involving overtime cost, changeover cost, etc.

Most of the research on the VCP is based on an integer model involving two types of variables corresponding respectively to vehicle and to crew duties. Vehicle variables are usually multicommodity flow type variables. Concerning the crews, the model only deals with feasible crew duties which, due to its huge number, are usually considered implicitly.

Several authors have addressed the single depot VCP, either using approximation methods such as Freling et al. (1999, 2003), or proposing exact methods such as Haase and Friberg (1999) and Haase et al. (2001). Note that, the single depot vehicle scheduling problem is polynomially solvable while the multi-depot problem, which is more general since it takes into account the different locations for the depots as well as the different types of vehicles, is NP-Hard. Most of the research done for the multi-depot case is based on relaxations of the integer model combined with a column generation scheme. For instance, Huisman et al. (2005), De Groot and Huisman (2004) and Borndörfer et al. (2004) use lagrangean relaxation while Mesquita and Paias (2008) use linear programming relaxation.

The drivers' rostering problem (DRP) in a public transit company consists of assigning the company's drivers to a set of crew duties that is adequate to operate all the vehicle blocks during the time horizon, also referred to as rostering period, while complying with specific constraints, besides management and worker preferences. The DRP is an NP-hard problem as proved for a similar rostering issue in Moz and Pato (2007).

Some companies follow procedures that divide the set of drivers into small groups scheduled independently on a cyclic basis. Other companies adopt procedures that assign the duties sequentially to each driver at a time, on the basis of seniority. The advantage of both approaches lies in the simplicity of the processes, although they are less adaptable to general conditions. Note that, when rostering on a cyclic basis the procedure is simple and seems to be fair, as following a number of rostering periods all workers have got the same work pattern and load. But this fairness is only apparent as in the real life situations absences and other occurrences arise, forcing cycles and consequently fairness to be impaired. Approaches that assign crew duties simultaneously to all drivers have been proposed mainly within other transport contexts (see, for instance, Kohl and Karish (2004) for railway and air crew rostering). In this situation, all rosters are considered and not only the subset of the cyclic ones.

In terms of the mathematical models for rostering in several transport contexts, including public transit companies, the most relevant models are based on multilayer networks or on set covering/partitioning models. Multilayer network models began with the seminal research of Carraresi and Gallo (1984) and has continued with more recent works by Aringhieri and Cordone (2004) for refuse collection staff and Cappanera and Gallo (2004) for air crews, while set covering/partitioning models emerge from the paper of Dantzig (1954) for rostering toll booths to more recent ones such as Freling et al. (2004) for railway and air crews and Catanas and Paixão (1995) specifically for bus driver rostering.

As for the methods developed to solve general rostering problems, one can refer lagrangean relaxation, column generation, branch-and-bound, constraint programming and heuristics; an extensive review can be found in Ernst el al. (2004). These methods hardly cope with some complex public transit situations, where the driver rostering constraints tend to increase in number and diversity within high dimension instances of the problem. Hence, new methodologies to efficiently tackle these real instances are urgently needed.

The wide variety of rostering constraints, together with the need to do as much as possible to reconcile potentially conflicting interests, leads naturally to the application of multi-objective optimization models.

To the authors' knowledge, only a few papers addressing multi-objective rostering problems have been published, among them, Catanas and Paixão (1995) and Moz et al. (2007) for bus driver; Caprara et al. (1998), Freling et al. (2004) and Lucic and Teodorovic (1999), all for train and/or air crews; and Chu (2007) for airport staff. Multi-objective models have not been appreciably devised to build driver rosters in public transit companies since the set of drivers operating the vehicles in a specific metropolitan area is usually homogeneous and fixed or easily determined, hence the only goal is to balance the workload. However, a multi-objective approach is required, for instance, if the pool of drivers is not fixed or is not homogeneous, for example in terms of costs, age or skills.

Note that theoretical research has been developed to support the DRP, albeit far less than on vehicle and crew scheduling. Ernst et al. (2004) published a survey of practical and theoretical research work on rostering up to 2004 and, since that date, few works concerning public transit companies have appeared (see Hartog et al. (2006), Lezaun et al. (2006) and Lucic and Teodorovic (2007)).

Finally, the problem addressed in this paper is the integrated vehicle-crew-rostering problem (VCRP) aiming to assign drivers of a public transit company, during a time horizon, to the vehicle blocks built according to a previously defined demand of passengers in a specific area. To the authors' knowledge, no other mathematical formulation or specific methodologies for the VCRP have been published.

3. A Mathematical Formulation

This section is devoted to the detailed description of VCRP and, simultaneously, to the presentation of a multi-objective binary programming formulation.

Given a time horizon *H*, the integrated vehicle-crew-rostering problem aims to simultaneously determine the set of vehicle blocks that cover all timetabled trips for a specific area, the set of crew duties that cover all vehicle blocks and the sequence of crew duties to be performed by each single crew/driver. The rostering period *H* is partitioned into planning periods, usually days, with an integer number of weeks, α, and the vehicle blocks as well as the crew duties are built for each day *h*, $h \in H = \{1,...,|H|\}$.

Let $T_1^h,...,T_n^h$ be a set of timetabled trips (trips, for short) to be operated in the day *h* by vehicles located at depots $D_1,...,D_k$. In depot D_d, $d = 1,...,k$, there are v_d identical vehicles. Each vehicle block starts and ends at the same depot. For each trip the starting and ending time and location are known and trips T_s^h and T_t^h are said to be compatible if the same vehicle can perform trip T_s^h and trip T_t^h in sequence. A deadhead trip is the movement of a vehicle without passengers. There are three types of deadhead trips: those between the end location of T_s^h and the start location of T_t^h, those from a depot to the start location of a trip and those from an end location of a trip to a depot. The second case is usually denoted by pull-out trips while the third is denoted by pull-in trips. A cost is assigned to each deadhead trip.

Each end location of a trip is a potential relief point, where it is allowed to replace drivers. Therefore, each task, the smallest amount of work to be assigned to the same vehicle and crew, corresponds to a deadhead trip followed by a trip, and the crew duties can start (end) at a depot or at an end location of a trip. A crew duty is a combination of tasks that respects labor and company rules. These rules depend on the particular situation in study and usually constraint the maximum and minimum spread, the maximum working time without a break, the break duration, etc.

A vertex is associated to each timetabled trip and the vertices are ordered by day of the rostering period, that is, considering first the timetabled trips corresponding to day h=1, then day h=2, and so on. For each day (period), the timetabled trips are ordered by increasing value of their starting time. Let $N^h = \left\{ \sum_{q=1}^{h-1} n_q + 1, \cdots, \sum_{q=1}^{h-1} n_q + n_h \right\}$, be the set of vertices associated to day h, where vertex $\sum_{q=1}^{h-1} n_q + i$ corresponds to trip T_i^h and $n_h=|N^h|$.

Let $D = \{n+1, \ldots, n+k\}$ be the set of vertices associated to the depots where $n = \sum_{h=1}^{H} n_h$ and vertex $n+d$ corresponds to depot d, $d = 1, \ldots, k$. For each $h \in H$ and each $d \in D$ a graph $G^{dh} = \left(V^{dh}, A^{dh}\right)$ is considered, where $V^{dh} = N^h \cup \{n+d\}$ and $A^{dh} = I^h \cup \left\{(n+d) \times N^h\right\} \cup \left\{N^h \times (d+n)\right\}$. The arc set A^{dh} contains arcs corresponding to pairs of compatible trips, $I^h \subseteq N^h \times N^h$, and arcs related with pull-in trips from depot d and pull-out trips from depot d.

If trips $s \in N^h$ are ordered by increasing value of their starting time then the arc set I^h contains only arcs (s,t) with $s < t$ and no circuit containing only vertices $s,t \in N^h$ exists in graph G^{dh} for any d.

Let L^h represent the set of feasible crew duties for day h. Let $L(s,t)$ denote the set of crew duties covering the deadhead trip from the end location of trip s to the start location of trip t and covering trip t. Let $DL(t)$ represent the set of duties covering the deadhead trip from any depot to the start location of trip t and covering trip t and let $LD(s)$ denote the set of duties covering the deadhead trip from the end location of trip s to any depot.

We define $I_c^h \subseteq I^h$ the subset of arcs where a changeover or a driver walking movement may occur.

Consider the decision variables respecting the vehicle and crew scheduling process:

$$z_{st}^{dh} = \begin{cases} 1 & \text{if a vehicle from depot } d \text{ performs} \\ & \text{trips } s \text{ and } t \text{ in sequence on day } h \\ 0 & \text{otherwise} \end{cases} \quad , \ (s,t) \in I^h, d \in D, h \in H;$$

$$z_{s,n+d}^{h} = \begin{cases} 1 & \text{if the bus returns to depot } d \\ & \text{after trip } s \text{ on day } h \\ 0 & \text{otherwise} \end{cases} \quad , \ s \in N^h, \ d \in D, \ h \in H;$$

$$z_{n+d,t}^{h} = \begin{cases} 1 & \text{if depot } d \text{ directly supplies} \\ & \text{a bus for trip } t \text{ on day } h \\ 0 & \text{otherwise} \end{cases} \quad , \ t \in N^h, \ d \in D, \ h \in H;$$

$$w_{\ell}^{h} = \begin{cases} 1 & \text{if crew duty } \ell \text{ is selected on day } h \\ 0 & \text{otherwise} \end{cases} \quad , \ \ell \in L^h, \ h \in H.$$

The DRP consists of assigning a set of crew duties covering the rostering period H to the drivers available to operate the company's vehicles. Here, one considers a line of work to be a sequence of crew duties or days-off, one per day, assigned to a particular driver during the rostering period. A roster is a set of lines of work for the drivers of the company that must satisfy the so-called hard constraints that follow:

– (hard1) each crew duty must be assigned to one and only one driver;
– (hard2) each driver must be assigned to one crew duty or to a day-off on each day;
– (hard3) drivers must rest a given minimum number of hours between consecutive duties (here, to impose this constraint, the set of crew duties is partitioned into early duties and late duties thus forbiding the sequence late-early duties);

– (hard4) drivers must work at the most a given number of hours per week and a given number of hours during the rostering period;

– (hard5) drivers must work at the most a given number of consecutive days;

– (hard6) drivers must have at least a given number of days-off per week;

– (hard7) some drivers must have specific weekdays off (for instance, due to planned absences or holidays) or specific weekends off (due to seniority);

– (hard8) drivers must have a given number of Sundays off in the rostering period.

However, a roster to be accepted in the company should also comply with other kind of requirements arising from the interests of the two main parties involved in the process: management and drivers. Here, two soft constraints are considered to be the most important requirements each party would like to achieve:

– (soft1) operate the vehicles of a given area during the rostering period with the minimum number of drivers;

– (soft2) do not assign overtime to drivers, otherwise, evenly distribute it among drivers.

The soft constraint (soft1) arises because management often wants to know the minimum workforce required to operate the fleet of vehicles, in order to assign drivers to other sectors of the company or to replace those absent. As overtime is undesirable for drivers, it should be minimized and equitably distributed (soft 2). These constraints (soft1) and (soft2) represent conflicting interests that normally cannot be simultaneously fulfilled; hence they will be formalized through objectives within the multi-objective model presented in the next paragraphs.

Let M be the set of drivers of the company and consider that L^h, the set of duties of day h, is partitioned into L^{h_1}, the set of early duties and L^{h_2}, the set of late duties. The following parameters are required for the rostering process:

u_ℓ = spread of crew duty ℓ (in hours), $\ell \in L^h$, $h \in H$;

$\overline{u}$ = normal working time of a crew duty;

u'_ℓ = max $\{0, \quad u_\ell - \overline{u}\}$, $\ell \in L^h$, $h \in H$;

b_{1w} = maximum total work per week per driver;

b_{rw} = maximum total work per rostering period per driver;

Ω_w = minimum number of days-off per week per driver;

Ω_S = minimum number of Sundays-off per rostering period per driver;

g = maximum number of consecutive days without a day-off;

$$e^{mh} = \begin{cases} 1 & \text{if driver } m \text{ was assigned to a crew duty on day } h \\ & \text{of the previous rostering period} \\ 0 & \text{otherwise} \end{cases} \quad, \ m \in M \text{ , } h = 1\text{-}g,\ldots,0;$$

F^m = set of days-off for driver m, during the rostering period, $m \in M$.

Here, the day-off is considered as an artificial duty represented by the label O^h, the $(|L^h|+1)^{th}$ "duty", for each day $h \in H$. Now, the decision variables relative to the rostering process are introduced:

$$y^{mh}_{\ell} = \begin{cases} 1 & \text{if driver } m \text{ performs duty} \\ & \text{(or day-off) } \ell \text{ on day } h \\ 0 & \text{otherwise} \end{cases} \quad, \ m \in M \text{ , } \ell \in L^h \cup \{O^h\}, \ h \in H \text{ .}$$

Then, the constraint set of VCRP becomes:

$$\sum_{d \in D} \sum_{t:(s,t) \in I^h} z^{dh}_{st} + \sum_{d \in D} z^h_{s,n+d} = 1 \qquad \forall s \in N^h, \forall h \in H \tag{3.1}$$

$$\sum_{t:(s,t) \in I^h} z^{dh}_{st} + z^h_{s,n+d} - \sum_{t:(t,s) \in I^h} z^{dh}_{ts} - z^h_{n+d,s} = 0 \quad \forall s \in N^h, \forall d \in D, \forall h \in H \tag{3.2}$$

$$\sum_{s \in N^h} z^h_{n+d,s} \le v_d \qquad \forall d \in D, \forall h \in H \tag{3.3}$$

$$\sum_{\ell \in DL(t)} w^h_\ell - \sum_{d \in D} z^h_{n+d,t} = 0 \qquad \forall t \in N^h, \forall h \in H \tag{3.4}$$

$$\sum_{\ell \in L(s,t)} w^h_\ell - \sum_{d \in D} z^{dh}_{st} = 0 \qquad \forall (s,t) \in I^h \setminus I^h_c, \forall h \in H \tag{3.5}$$

$$\sum_{\ell \in L(s,t)} w^h_\ell - \sum_{d \in D} z^{dh}_{st} \ge 0 \qquad \forall (s,t) \in I^h_c, \forall h \in H \tag{3.5'}$$

$$\sum_{\ell \in LD(s)} w^h_\ell - \sum_{d \in D} z^h_{s,n+d} = 0 \qquad \forall s \in N^h, \forall h \in H \tag{3.6}$$

$$\sum_{m\in M} y_\ell^{mh} - w_\ell^h = 0 \qquad \forall \ell \in L^h, \forall h \in H \qquad (3.7)$$

$$\sum_{\ell\in L^h \cup \{O^h\}} y_\ell^{mh} = 1 \qquad \forall m \in M, \forall h \in H \qquad (3.8)$$

$$\sum_{\ell\in L^{h_1}} y_\ell^{mh} + \sum_{\ell\in L^{h_2}} y_\ell^{mh} \le 1 \qquad \forall m \in M, h = 2,\ldots,|H| \qquad (3.9)$$

$$\sum_{\ell\in L^h} \sum_{h=7(l-1)+1}^{7l} u_\ell \; y_\ell^{mh} \le b_{1w} \qquad \forall m \in M, l = 1,\ldots,\alpha \qquad (3.10)$$

$$\sum_{\ell\in L^h} \sum_{h\in H} u_\ell \; y_\ell^{mh} \le b_{rw} \qquad \forall\, m \in M \qquad (3.11)$$

$$\sum_{\ell\in L^h} \sum_{r=0}^{g} y_\ell^{m,h+r} \le g \qquad \forall m \in M,\, h = 1,\ldots,|\,H\,| - g \qquad (3.12)$$

$$\sum_{r=h}^{0} e^{mr} + \sum_{\ell\in L^h} \sum_{r=1}^{h+g} y_\ell^{mr} \le g \qquad \forall m \in M,\, h = 1-g,\ldots,-1,0 \qquad (3.13)$$

$$\sum_{h=7(l-1)+1}^{7l} y_{O^h}^{mh} \ge \Omega_W \qquad \forall m \in M,\, l = 1,\ldots,\alpha \qquad (3.14)$$

$$\sum_{\ell\in L^h} y_\ell^{mh} = 0 \qquad \forall m \in M,\, h = 1,\ldots,|H| \text{ and } h \in F^m \qquad (3.15)$$

$$\sum_{l=1}^{\alpha} y_{O^h}^{m,7l} \ge \Omega_S \qquad \forall m \in M \qquad (3.16)$$

$$z_{ij}^{dh} \in \{0,1\} \qquad \forall (s,t) \in I^h, \forall d \in D, \forall h \in H \qquad (3.17)$$

$$z_{s,n+d}^h,\, z_{n+d,s}^h \in \{0,1\} \qquad \forall s \in N^h, \forall d \in D, \forall h \in H \qquad (3.18)$$

$$w_\ell^h \in \{0,1\} \qquad \forall \ell \in L^h, \forall h \in H \qquad (3.19)$$

$$y_\ell^{mh} \in \{0,1\} \qquad \forall m \in M, \ell \in L^h \cup \{O^h\}, \forall h \in H. \quad (3.20)$$

Constraints (3.1), (3.2) and (3.3) describe the vehicle scheduling problem. Constraints (3.1) state that, for each day, each timetabled trip is performed exactly once. Constraints (3.2) ensure that each vehicle block starts and ends at the same depot. Constraints (3.3) are depot capacity constraints. The constraint set (3.4)-(3.6) links vehicle and crew variables ensuring that each arc in a vehicle block is covered by precisely one crew. Constraints (3.5)' allow, on one hand, that an arc in I_c^h may be covered by more than one crew and, on the other hand, that it may be covered by a crew without being covered by a vehicle. This last situation corresponds to a changeover, that is, to the walking movement of a driver between two timetabled trips in order to change the vehicle.

Equalities (3.7) and (3.8) impose the rostering hard constraints (hard1) and (hard2). As to (3.7), by ensuring that there is always a driver covering each crew duty, it relates the crew scheduling and roster variables. Now, rostering hard constraint (hard3) is imposed by (3.9) assuming that the previously defined minimum number of resting hours forbids the sequence of late duty followed by early duty, as mentioned above. Additional inequalities (3.10) and (3.11) force hard constraint (hard4). Furthermore, the inequalities (3.12)-(3.16) impose hard constraints (hard5), (hard6), (hard7) and (hard8) for the rostering process.

Finally, conditions (3.17)-(3.20) express the domains of the variables.

Note that, assuming prior knowledge of the set of crew duties, that is, the values for the variables w_ℓ^h (see equalities (3.7)), the solution of the system of linear (in)equalities (3.7)-(3.16) and (3.20) defines a feasible roster for DRP, that is, one that satisfies all hard constraints.

Next, the objective functions of the VCRP are formalized. As for the vehicles, costs c_{st}^{dh}, $c_{s,n+d}^{h}$ and $c_{n+d,t}^{h}$ are associated with the corresponding arcs of each graph $G^{dh} = \left(V^{dh}, A^{dh}\right)$. Concerning the crews, a cost s_ℓ is assigned to each crew duty ℓ.

New variables are necessary to formalize the objectives of the rostering:

$$\omega^m = \begin{cases} 1 & \text{if driver } m \text{ works during the rostering period} \\ 0 & \text{otherwise} \end{cases}, \quad m \in M.$$

By joining the previous system of linear (in)equalities with the mathematical representation of the optimization objectives of vehicle scheduling, crew scheduling and rostering, the multi-objective mathematical formulation for the integrated vehicle-crew-rostering problem arises:

min

$$fvc_0 = \sum_{h \in H} \left(\sum_{d \in D} \sum_{(i,j) \in I} c_{ij}^{dh} z_{ij}^{dh} + \sum_{d \in D} \sum_{i \in N} \left(c_{i,n+d}^{h} z_{i,n+d}^{h} + c_{n+d,i}^{h} z_{n+d,i}^{h} \right) + \sum_{\ell \in L^h} s_\ell w_\ell^h \right) \tag{3.21}$$

$$\min \quad fr_1 = \sum_{m \in M} \omega^m \tag{3.22}$$

$$\min \quad fr_2 = \max_{m \in M} \sum_{\ell \in L^h} \sum_{h \in H} u'_\ell \, y_\ell^{mh} \tag{3.23}$$

subject to

(3.1)-(3.20)

$$\sum_{\ell \in L^h} \sum_{h \in H} y_\ell^{mh} - |H| \omega^m \leq 0 \qquad \forall m \in M \tag{3.24}$$

$$\omega^m \in \{0,1\} \qquad \forall m \in M. \tag{3.25}$$

This is a multi-objective binary programming problem where min fvc_0 aims to minimize the costs of vehicle and crew scheduling. The other objectives, minimization of fr_1 and fr_2, model the two driver rostering soft constraints: (soft1), minimize number of drivers assigned to work, and (soft2) minimize the overtime per driver, during the rostering period. Constraints (3.24) link variables ω^m with rostering variables y_ℓ^{mh}.

In summary terms, the VCRP was modeled within a multi-objective linear programming problem, with three objectives.

4. Solution Approach

The solution methodology devised for the VCRP is a preemptive goal programming approach.

A slack variable δ^{+}, denoting the maximum overtime per driver during the rostering period, is introduced to linearize fr_2 and the goal programming model follows:

$$\min \ \lambda_0 \underbrace{\sum_{h\in H}\left(\sum_{d\in D}\sum_{(i,j)\in I} c_{ij}^{d} z_{ij}^{dh} + \sum_{d\in D}\sum_{i\in N}\left(c_{i,n+d} z_{i,n+d}^{h} + c_{n+d,i} z_{n+d,i}^{h}\right) + \sum_{\ell\in L^h} s_\ell w_\ell^h\right)}_{fvc_0}$$

$$+$$

$$+\lambda_1 \underbrace{\sum_{m\in M}\omega^m}_{fr_1} + \lambda_2 \underbrace{\delta^{+}}_{fr_2} \qquad (4.1)$$

subject to

(3.1)-(3.20), (3.24) and (3.25)

$$\sum_{\ell\in L^h}\sum_{h\in H} u'_\ell \, y_\ell^{mh} - \delta^{+} \le 0 \qquad \forall m \in M \qquad (4.2)$$

$$\delta^{+} \ge 0 \qquad (4.3)$$

where λ_0 is set to a big penalty, and $\lambda_1, \lambda_2 \in [0,1]$ are the goals' parameters.

This is a preemptive goal programming problem with a higher priority goal (associated with λ_0) forcing minimization of vehicle plus crew scheduling costs, represented by function fvc_0 in (4.1). At a lower priority level, two other goals (minimization of fr_1 and fr_2) modelize the two driver rostering soft constraints, (soft1) and (soft2) respectively, and all mixed strategies obtainable by changing the parameters λ_1 and λ_2, in the specified interval, see again (4.1). Here, the conditions (4.2) and (4.3) define the goal programming variable for the third goal of the problem.

Note that, when fixing the three penalizing parameters, λ_0, λ_1 and λ_2, the above formulation becomes a mixed binary linear programming problem (MILP) with a single continuous variable, δ^{+}.

As a first step, the solving approach minimizes fvc_0 subject to (3.1)-(3.6) and (3.17)-(3.19), that is, a VCP is solved producing the vehicle blocks defined by the values of variables z_{st}^{dh}, $z_{s,n+d}^{h}$, $z_{n+d,t}^{h}$, and the crew duties defined by the values of variables w_{ℓ}^{h}. This problem can be split into |*H*| independent integrated vehicle and crew scheduling problems, one for each day *h* with $h = 1,...,|H|$. As a second step, to produce the roster for the rostering period, that is, to solve the DRP, the approach takes the values of the crew duty variables w_{ℓ}^{h} (already calculated) and minimizes $\lambda_1 fr_1 + \lambda_2 fr_2$ subject to (3.7)-(3.16), (3.20), (3.24), (3.25), (4.2) and (4.3), where λ_1 and λ_2 are suitably chosen within the domain.

4.1. Integrated vehicle and crew scheduling problem

The integrated vehicle and crew scheduling problem is solved with the approach proposed by Mesquita and Paias (2008). The method consists of three phases: first, preprocessing; second, solving the linear relaxation; third, obtaining a feasible solution.

In the first phase the set of tasks is defined and an initial set of crew duties is obtained. Tasks are determined by merging some pairs of compatible trips that, due to their starting and ending times and places, are expected to be covered by the same vehicle and crew. A predefined parameter ε is used to obtain the potential pairs of trips to merge. Two trips *i* and *j* may be merged whenever the elapsed time between the end of trip *i* and the start of trip *j* is less than ε. To decide which trips are effectively merged into tasks a multi depot vehicle scheduling problem without requiring that each vehicle returns to the source depot is solved. Note that this vehicle scheduling problem is easily solved by a polynomial type algorithm; see Mesquita and Paixão (1992).

In the second phase, the linear programming relaxation of the model is solved using a column generation scheme where the vehicle variables are explicitly considered while the crew variables are implicitly considered.

Each feasible crew duty corresponds to a path in an adequate network. The feasibility of the duties is established through the network definition, by including or excluding some arcs, and also by defining resources that are consumed along the network. The resource consumption is restricted by imposing time windows on some vertices. A path that respects the resources consumption corresponds to a feasible duty. The pricing problem is a shortest path problem with resource constraints. It is solved by a heuristic procedure that uses dynamic programming where states are associated to crew duties and the stages to tasks, and works with a reduced state space. A random factor, p, defined by the user, controls the complexity of the crew duty generator by reducing the number of duty candidates to be generated. In each stage of the dynamic program, each state is discarded with probability p. Note that, if a relevant state is discarded at a specific iteration, then it is expected to be generated in subsequent iterations and not always discarded. The random choice of states to be discarded leads to a smaller but varied final crew duty set, maintaining the quality of the linear relaxation bound but reducing the CPU time. A complete description of the duty generator is given in Mesquita and Paias (2008).

Finally, in the third phase, whenever the solution of the linear programming relaxation is not integer, an integer solution is obtained by branching on the set of crew duties generated while solving the linear relaxation.

It should be observed that a skilled management of the set of parameters is crucial for a successful integration between the VCP and the DRP.

4.2 Driver rostering problem

As mentioned above, if the integrated vehicle and crew scheduling is previously solved, the crew duties required per day within the rostering period are known – information given by variables w_ℓ^h – and the goal programming model (3.1)-(3.20), (3.24), (3.25) and (4.1)-(4.3) reduces to a sub-model corresponding to the DRP decisions. By fixing the parameters λ_1 and λ_2 this problem becomes a MILP that may be solved with an exact algorithm from a standard software package.

So as to enhance the computing efficiency of that exact algorithm, a trivial lower bound for the minimum number of drivers ($\underline{fr_1} \leq \text{minimum of } fr_1$) was calculated according to the formula:

$$\underline{fr_1} = \left\lceil \frac{\sum_{h \in H} | L^h |}{| H | - \alpha \, \Omega_w} \right\rceil \tag{4.4}$$

where the total number of crew duties for the rostering period is divided by the maximum number of work days of a driver per rostering period and, finally, rounded up if not integer. The expression (4.4) was deduced on the basis of equal transport demand during the weekdays and also equal demand during the weekends (Saturdays and Sundays). Note that, the MILP formulation with this trivial bound constraint led, in fact, to significant improvements in computational efficiency.

5. Computational Experiment

A computational experiment was run with data from a bus company in Lisbon where the fleet of vehicles is split among four depots. Five test problems of VCRP, with 122, 168, 224, 226 and 238 timetabled trips per weekday have been analyzed.

5.1 Test Data and Implementation

In terms of the VCP, the algorithms were coded in C, using VStudio 6.0/C++. The linear programming relaxation was solved using CPLEX 9.0. In order to minimize the number of operating vehicles in the schedule, a fixed cost of 10000 m.u. has been assigned to each vehicle, which is reached by adding a penalty equal to 5000 m.u. to the cost of each pull-in and each pull-out arc. Also, in order to minimize the number of crew duties to be covered by the drivers in each day, a fixed cost of 5000 m.u. has been assigned to each crew duty in the solution. This corresponds to set $s_\ell = 5000$ m.u., $\forall \ell \in L^h, h \in H$, in the objective function of the VCP.

According to some company rules, for each crew duty the minimum spread is set at 60 minutes. The maximum spread is 5 hours for duties without a break and 10 hours and 45 minutes, in other cases. Break times range from 60 to 140 minutes. The maximum duration allowed for a crew duty before a break occurs is 5 hours. In terms of the current DRP, the rosters must satisfy the rules imposed by Portuguese Law, union contracts and specific rules of the company. Hence, the resulting instances have $|H|$=28, that is, a rostering period of 28, consequently $\alpha = 4$.

The values for the following parameters are fixed at:

$u_\ell \in [300, 645]$ minutes, the spread of crew duties;

$\bar{u}$ = 480 minutes (8 hours), "normal" daily working time of crews;

a = 11 hours, minimum rest period between consecutive work duties, always respected when the sequence late-early is forbidden;

b_{1w} = 2880 minutes (48 hours), maximum total work per week per driver;

b_{rw} = 10560 minutes (176 hours), maximum total work per rostering period per driver;

Ω_w = 2, minimum number of days-off per week per driver;

Ω_S = 1, minimum number of Sundays-off per rostering period per driver;

g = 6 days, maximum number of consecutive workdays;

$F^m = \varnothing$, no compulsive days-off.

The bus transport system in this company is planned with two different demand patterns, one for weekdays and the other for weekends. Moreover, as referred to in section 3, the minimum rest period of 11 hours allows the separation of the set of crew duties into two types: the early duties, starting at a point between 6:00 a.m. and 3:30 p.m. and the late duties, starting in the interval 3:30 p.m. to midnight.

To solve the DRP instances, the CPLEX version 11 optimizer was applied to the MILP formulation pointed out in subsection 4.2.

5.2 Computational Results

The method proposed in section 4 was used to obtain a set of solutions for each one of the five test problems. The reasons for obtaining multiple solutions were on the one hand, to explore different strategies to deal with the multi-objective nature of the problem and, on the other hand, to evaluate the correlation between the quality of the VCP solutions and the quality of the rostering solutions.

During the optimizing process, two parameters are controlled by the user: ε for the adjustment of the tasks length; p for the trade-off between the accuracy of the pricing problem and the time spent to solve the linear programming relaxation. It is relevant to study the adjustment of the values of these parameters to determine good solutions for the VCP. Therefore, different values for ε have been combined with different values for p. The computational results were obtained on a PC Pentium IV 3.2 GHz and are presented in Figures 1 and 2.

For each test problem (122, 168, 224, 226 and 238) three different values of ε (5, 7 and 10 minutes) were considered. For each value of ε, three different values of p (1/10, 2/15, 1/6) were tested. Figure 1 shows the effect of the different parameter values on the number of vehicles and crews in the final solution, whereas Figure 2 shows the effect of the different parameter values on the CPU times (in seconds).

Figure 1: Total number of vehicles+crews for different values of p and ε

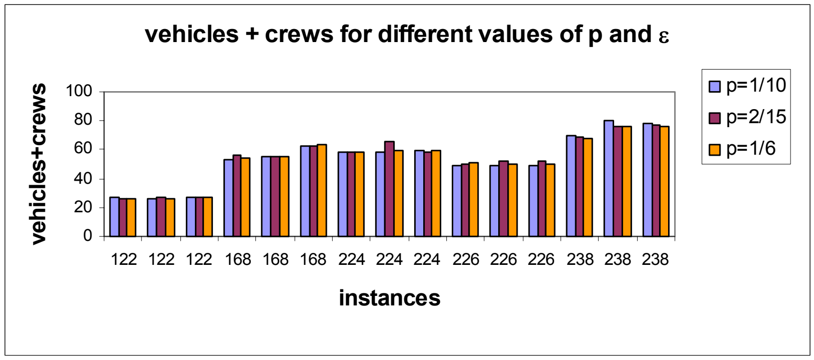


Figure 2: Computing time for different values of p and ε

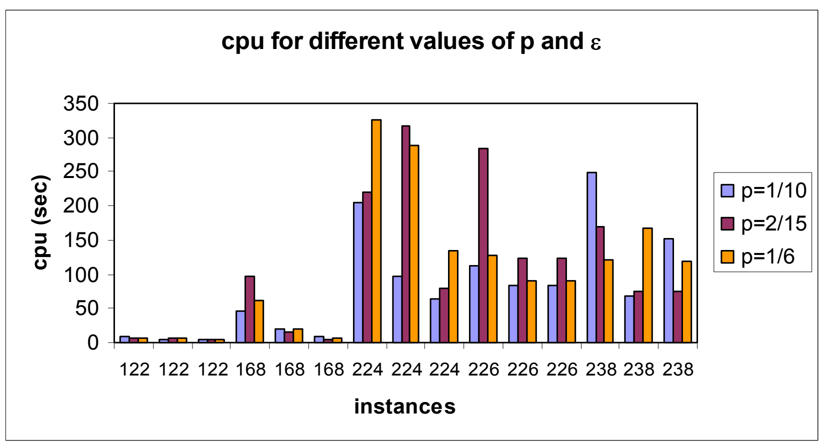


Figure 1 shows that the quality of the VCP solutions did not change for the different combinations of parameter values. However, Figure 2 reveals that a fine tuning of the parameters can greatly reduce the CPU time.

The set of crew duties given by each final VCP solution is the input data for the corresponding DRP instance. The DRP programs ran on a PC Pentium IV Dual Core, 2 Duo 1.8 GHz. For each one of the resulting 45 DRP instances, three single objective problems corresponding to three different rostering strategies were solved. In the first, $\lambda_1 = 1$ and $\lambda_2 = 0$, that is, the DRP with the single objective of minimizing the total number of drivers (minimize fr_1 in (4.1)), whose computational results are shown in Table 1. In the second, $\lambda_1 = 0$ and $\lambda_2 = 1$ in order to minimize the maximum overtime per driver during the rostering period (minimize fr_2 in (4.1)) and the corresponding results are presented in Table 2. Finally, Table 3 presents results obtained with the third strategy, $\lambda_1 = 0.96$ and $\lambda_2 = 0.04$. These values balance the weighted value of both optimization objectives for rostering, since the rate of the single objective values is about 1 to 24, for some representative instances.

For the three tables, columns (1)-(5) retain the same meaning and values, summarizing the features of the instances. Columns (1), (2) and (3) identify each DRP instance depending on the VCRP problem data and on parameters of the VCP solving process. Column (4) gives the number of crew duties per weekday (from the solution of the VCP), the number of crew duties per weekend day (determined on the assumption that in the area under study the timetables at weekends cover neither early morning nor late evening trips) and the total number of crew duties for the rostering period calculated by using the formula $5\alpha\left|L^1\right| + 2\alpha\left|L^6\right|$, which has already been used in (4.4) for the general case. Column (5) shows the value of the trivial lower bound for the minimum number of drivers, calculated from (4.4).

In Tables 1 to 3, columns (6) and (7) present the optimal value of the MILP formulation's linear relaxation and the respective CPU time in seconds. Columns (8) to (10) show, respectively, the MILP optimal value or the best upper bound when the solver did not stop with the optimal solution (two stopping criteria were established: a limit of four CPU hours and an "absolute mip" gap of 1 unit), the relative gap in percentage (optimum or upper bound minus CPLEX last lower bound divided by this lower bound, times 100) and the CPU time in seconds. Column (11) displays the maximum overtime assigned to a driver during the rostering period (in minutes) and column (12) the number of drivers assigned to work during the rostering period. The values in both columns are calculated on the basis of a specific roster as follows:

- in Table 1, column (11), each value is obtained by minimizing fr_2 subject to (3.1)-(3.20), (3.24), (3.25), (4.2) and (4.3), as well as to an extra constraint forcing fr_1 to be equal to the minimum given in column (8) (in this table, column (12) is equal to column (8));

- in Table 2, column (12), each value is obtained by minimizing fr_1 subject to (3.1)-(3.20), (3.24), (3.25), (4.2) and (4.3), as well as to an extra constraint forcing fr_2 to be less or equal to the minimum or upper bound given in column (8) (in this table, column (11) is equal to column (8));
- in Table 3, columns (11) and (12) show the figures for fr_2 and fr_1 calculated from a roster that corresponds to the weighted value in column (8).

Note that, considering the DRP as a bi-objective optimization problem, columns (12) and (11) of the three tables indicate the coordinates of candidates to be non-dominated points. In the case of Tables 1 and 2, these are the candidate lexicographic points of the Pareto frontier. All these points are just candidates, without the guarantee of being non-dominated points, because optimality stopping condition was not attained when minimizing fr_2.

(Insert Table 1 here)

(Insert Table 2 here)

(Insert Table 3 here)

The first observation that arises from the figures concerns the quality of the trivial lower bound for the number of drivers assigned to work. As seen in columns (5) and (8) of Table 1, the best value was found for all the 45 instances. This happens with the current test problems, which is not only due to the low dimension workforce required at weekends but also to the fact that the hard constraints are not very restrictive, which in other real rostering situations may not occur. However, the inclusion of this additional constraint improved the linear lower bound and significantly reduced the computing time.

As, for the linear lower bounds (column 6), in Table 1, all except one are equal to the corresponding optimal values (column 8). In the other cases, Tables 2 and 3, the linear lower bounds have a higher gap in relation to the best known upper bound.

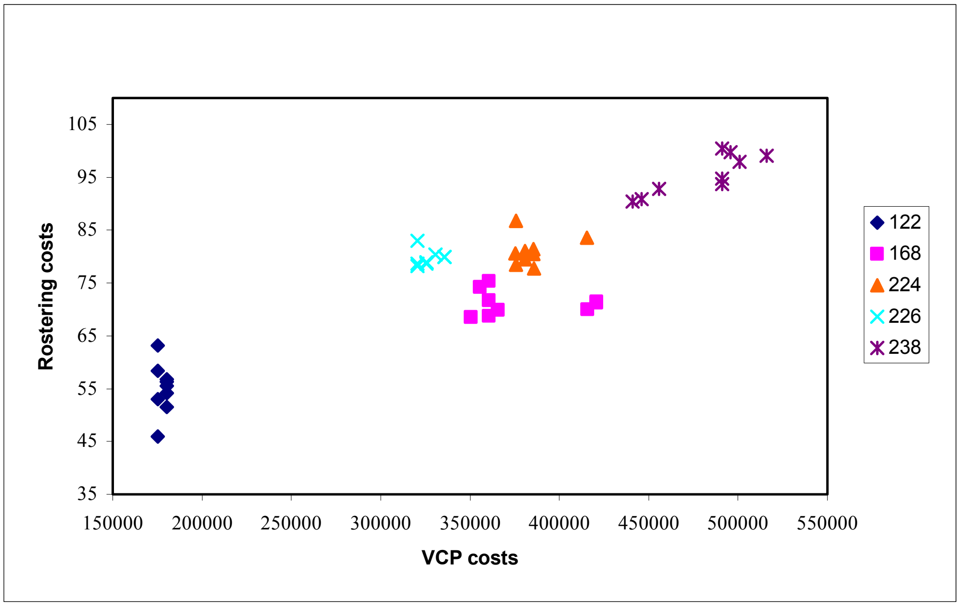
From these experiments one can observe that, for each instance, rosters satisfying all the hard constraints (feasible rosters) were found in keeping with different rostering strategies (from the three choices for λ_1 and λ_2), in reasonable computing time. The quality of the rosters in terms of soft constraints, that is, the satisfaction of (soft1) and (soft2) can be seen in columns (12) and (11), respectively.

The optimal solutions were easily found in short computing times in the case of minimizing the number of drivers alone (see, Table 1, column (10)). But, when optimizing the maximum overtime per driver, either alone (λ_1=0 and λ_2=1) or combined with the workforce dimension (λ_1=0.96 and λ_2=0.04), the computing efficiency was lower, as may be seen in Tables 2 and 3 (columns (8) and (10)). The same happened also when minimizing fr_2 subject to a fixed value for fr_1, see column (11) of Table 1. In fact, all these 45 MILP instances were not optimally solved by the CPLEX algorithm due to excessive computing time (time limit of 4 CPU hours). Respecting the optimization of fr_1 subject to an upper bound on the value of fr_2 (column (12) of Table 2) 17.8% instances were not solved within the time limit and each one of the remaining was solved, on average, in 5741.8 seconds. It is also interesting to note that the computing time necessary to solve the linear relaxation is not significantly higher for these difficult instances (see columns (7) in Tables 2 and 3) than it is for the easy cases (column (7), Table 1).

To sum up, the results displayed on Tables 1 and 2 confirm the conflicting nature of the two rostering objectives. As expected, the optimization of one objective can only be obtained at the expense of the other. On the other hand, Table 3 shows that it is possible to produce rosters that reconcile the interests of both agents involved in the rostering process. However, in using the sequential process for the VCRP, whether a single solution or a set of different solutions is produced by the VCP algorithm, as in the experiment under study (nine VCP solutions per VCRP problem), the objectives of rostering may not be satisfactorily contemplated. In fact, as some statistical tests will show next, there is no correlation between the quality of each VCP solution (fvc_0) and the quality of the corresponding DRP solution ($\lambda_1 fr_1 + \lambda_2 fr_2$). Consequently, a complete integration should be considered in the future.

For each of the 45 instances Figure 3 plots the vehicle and crew costs versus the rostering costs whilst, for each test problem Table 4 presents the corresponding correlation coefficient. The penalizing parameters λ_1 and λ_2 for the rostering problem, were set to λ_1=0.96 and λ_2=0.04.

Figure 3: Vehicle and crew cost versus rostering costs (λ_1=0.96 and λ_2=0.04)



(Insert Table 4 here)

From Figure 3 and Table 4, one may conclude that there is no correlation between the vehicle-crew costs and the rostering costs. In fact, excepting problem 238, the absolute values of the correlation coefficients range from 0.0263 to 0.2089.

Even when one considers just one of the objectives, there is no pattern for the values of the correlation coefficients as shown on Tables 5 and 6.

(Insert Table 5 here)

(Insert Table 6 here)

In conclusion, given a new problem, one is unable to predict if "good" solutions for the VCP will lead to good solutions for the DRP. The only way to guarantee low vehicle costs and good rosters is to solve the integrated vehicle-crew-roster problem.

6. Conclusions and Future Research

This paper presents an integrated formulation modeling public transit scheduling problems. The vehicle schedule, the drivers' schedule, as well as the rosters for the drivers are obtained concurrently as opposed to being separately determined, as is usually the case in other approaches.

It is well known that scheduling on public transit companies encompasses several combinatorial optimization problems that are strongly interconnected and can be hierarchically organized. The solution to each problem provides information for the others. In fact, the vehicle schedule (definition of the vehicle blocks) supports the definition of crew duties (work days) which, in turn, must be assigned to drivers, over the rostering period, which leads to a roster. Due to their individual complexity, the problems mentioned are usually handled separately and in sequence, possibly leading to poor quality final solutions. As rostering relies on the output of the other two schedules, one cannot guarantee that the final result is the best solution to the overall problem.

Typically, rostering applies to a period including several VCP planning periods (or days) and, due to its larger extent, it must combine sequencing constraints with a broader spectrum. Furthermore, an attempt to anticipate some of these constraints, while solving the VCP, may lead to non-optimal global solutions. For instance, take the case of a set of crew duties coming from the VCP, some very long, others very short, each one to be assigned to a single driver during a day. Perhaps, if timetabled trips and/or tasks were combined in another way, eventually leading to worse results for vehicle and crew optimization, from the rostering and even from the global VCRP standpoint, they would produce better solutions. In fact, crew duties arrive at the rostering phase as data, with no possibility of adjustments.

Previous experience has revealed that an integrated approach for the vehicle and crew scheduling problems is accomplishable and provides high quality solutions, Mesquita and Paias (2008) and Mesquita et al. (2006) thus pointing to an extension to accommodate rostering decisions as well. As a result, this paper proposes a multi-objective mixed binary linear programming formulation for the integrated vehicle-crew-rostering problem, modeling the different decisions and constraints arising in the operational planning phase in public transit companies. This monolithic model is approached by solving the integrated vehicle and crew scheduling problem for different values of the VCP parameters, hence producing a diverse set of final solutions. Afterwards, the resulting rostering problems are solved. One such approach may be the basis of a decision support system able to indicate to the user the effects of parameters in the final solutions and, at the same time, to produce a diversity of vehicle schedules and rosters.

Moreover, it should be emphasized that the driver rostering problem analyzed in this paper potentially has a wide range of applications. As we have seen, it deals with a planning situation where rest periods are imposed, where different durations for the rostering period may be considered, where one must take into account the tasks performed by the drivers beforehand, in the last days of the previous period, when one allows for different demand patterns of transport from day to day, and where a multitude of other scheduling rules can be introduced. Within this process, the rostering is naturally guided by two optimization objectives: minimization of driver costs and balancing of overtime work, representing the interests of the management and of the drivers of the company respectively. Note that not only public transit companies but also other transport companies share similar requirements as regards the rostering process.

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Figure legends

Figure 1: Total number of vehicles+crews for different values of p and ε

Figure 2: Computing time for different values of p and ε

Figure 3: Vehicle and crew cost versus rostering costs (λ_1 =0.96 and λ_2 =0.04)

Tables

Table 1: Computational results from DRP problems with $\lambda_1 = 1$ and $\lambda_2 = 0$

(1)	(2)	(3)	(4)			(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
instances	ε	*p*	nb crew duties			trivial lower	linear lower	CPU (sec)	optimal or upper	% gap	CPU (sec)	δ+ (min)	nb drivers
122	5	1/10	17	11	428	22	22	0.39	22	0	9.38	765	22
		2/15	17	11	428	22	22	0.38	22	0	38.00	812	22
		1/6	17	11	428	22	22	0.38	22	0	2.77	1097	22
	7	1/10	17	11	428	22	22	0.31	22	0	2.70	965	22
		2/15	17	11	428	22	22	0.36	22	0	4.45	832	22
		1/6	17	10	420	21	21	0.3	21	0	5.08	644	21
	10	1/10	18	12	456	23	23	0.44	23	0	4.42	870	23
		2/15	18	11	448	23	23	0.39	23	0	4.89	849	23
		1/6	18	12	456	23	23	0.45	23	0	3.41	867	23
average					435.56	22.22	22.22	0.38	22.22	0.00	8.34	855.67	22.22
168	5	1/10	36	18	864	44	44	1.89	44	0	17.83	665	44
		2/15	36	17	856	43	43	1.84	43	0	30.70	764	43
		1/6	37	19	892	45	45	2.33	45	0	106.99	783	45
	7	1/10	38	19	912	46	46	2.17	46	0	12.63	779	46
		2/15	38	18	904	46	46	2.31	46	0	12.42	613	46
		1/6	38	17	896	45	45	2.06	45	0	20.92	722	45
	10	1/10	42	20	1000	50	50	2.58	50	0	296.81	566	50
		2/15	42	20	1000	50	50	2.41	50	0	45.59	611	50
		1/6	42	20	1000	50	50	2.59	50	0	44.48	609	50
average					924.89	46.56	46.56	2.24	46.56	0.00	65.37	679.11	46.56
224	5	1/10	41	18	964	49	49	2.89	49	0	101.41	868	49
		2/15	41	20	980	49	50	3.05	50	0	468.44	1047	50
		1/6	41	19	972	49	49	2.83	49	0	79.73	814	49
	7	1/10	40	23	984	50	50	3.03	50	0	47.67	832	50
		2/15	39	21	948	48	48	2.75	48	0	333.98	982	48
		1/6	41	21	988	50	50	2.88	50	0	262.91	767	50
	10	1/10	41	21	988	50	50	3.19	50	0	183.50	664	50
		2/15	40	21	968	49	49	3.02	49	0	27.27	880	49
		1/6	41	22	996	50	50	2.95	50	0	87.86	846	50
average					976.44	49.33	49.44	2.95	49.44	0.00	176.97	855.56	49.44
226	5	1/10	34	18	824	42	42	2.58	42	0	207.53	1058	42
		2/15	35	19	852	43	43	2.06	43	0	303.50	966	43
		1/6	36	19	872	44	44	2.49	44	0	59.28	952	44
	7	1/10	34	17	816	41	41	2.61	41	0	316.89	966	41
		2/15	37	19	892	45	45	3.49	45	0	22.53	917	45
		1/6	35	18	844	43	43	1.94	43	0	149.13	945	43
	10	1/10	34	17	816	41	41	1.72	41	0	268.28	989	41
		2/15	37	19	892	45	45	3.51	45	0	22.33	915	45
		1/6	35	18	844	43	43	1.95	43	0	149.48	947	43
average					850.22	43.00	43.00	2.48	43.00	0.00	166.55	961.67	43.00
238	5	1/10	49	26	1188	60	60	12.94	60	0	14.70	871	60
		2/15	49	25	1180	59	59	11.02	59	0	21.94	862	59
		1/6	48	25	1160	58	58	6.55	58	0	17.03	890	58
	7	1/10	57	33	1404	71	71	16.33	71	0	29.05	781	71
		2/15	54	30	1320	66	66	10.03	66	0	53.28	931	66
		1/6	54	30	1320	66	66	9.09	66	0	42.38	789	66
	10	1/10	56	32	1376	69	69	15.59	69	0	28.72	796	69
		2/15	55	31	1348	68	68	14.31	68	0	33.28	867	68
		1/6	54	30	1320	66	66	9.59	66	0	63.23	807	66
average					1290.67	64.78	64.78	11.72	64.78	0.00	33.73	843.78	64.78

Table 2: Computational results from DRP problems with $\lambda_1 = 0$ and $\lambda_2 = 1$

(1)	(2)	(3)	(4)			(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
instances	ε	*p*	nb crew duties			trivial lower	linear lower	CPU (sec)	optimal or upper	% gap	CPU (sec)	δ+ (min)	nb drivers
122	5	1/10	17	11	428	22	669.28	1.30	673	0.45	14400.28	673	25
		2/15	17	11	428	22	710.72	1.33	712	0.14	3241.86	712	25
		1/6	17	11	428	22	960.48	1.40	965	0.42	14400.03	965	25
	7	1/10	17	11	428	22	842.40	1.27	847	0.47	14400.03	847	25
		2/15	17	11	428	22	728.16	1.48	733	0.55	14400.02	733	25
		1/6	17	10	420	21	535.68	0.48	540	0.75	14400.11	540	25
	10	1/10	18	12	456	23	796.16	1.63	800	0.38	14400.03	800	25
		2/15	18	11	448	23	774.40	1.50	781	0.77	14400.02	781	25
		1/6	18	12	456	23	786.88	1.71	790	0.38	14400.06	790	25
average					435.56	22.22	756.02	1.34	760.11	0.48	13160.27	760.11	25.00
168	5	1/10	36	18	864	44	564.08	3.41	566	0.18	2758.08	566	50
		2/15	36	17	856	43	604.08	3.59	607	0.33	14400.14	607	50
		1/6	37	19	892	45	683.84	3.58	686	0.29	14400.14	686	50
	7	1/10	38	19	912	46	698.56	3.61	702	0.43	14400.68	702	50
		2/15	38	18	904	46	546.48	3.66	549	0.37	14400.13	549	50
		1/6	38	17	896	45	626.40	3.11	630	0.48	14400.30	630	50
	10	1/10	42	20	1000	50	449.53	3.56	454	0.89	14400.16	454	50
		2/15	42	20	1000	50	476.20	3.52	484	1.47	14411.55	484	60
		1/6	42	20	1000	50	481.67	3.38	489	1.45	14400.13	489	60
average					924.89	46.56	570.09	3.49	574.11	0.65	13107.92	574.11	52.22
224	5	1/10	41	18	964	49	681.67	3.95	685	0.44	14400.86	685	60
		2/15	41	20	980	49	800.40	4.88	803	0.25	14400.19	803	60
		1/6	41	19	972	49	638.27	3.94	640	0.16	4157.97	640	60
	7	1/10	40	23	984	50	651.80	4.36	654	0.31	14400.20	654	60
		2/15	39	21	948	48	745.47	3.70	750	0.54	14400.20	750	60
		1/6	41	21	988	50	614.40	4.30	617	0.33	14400.14	617	60
	10	1/10	41	21	988	50	691.93	3.83	694	0.29	14400.13	694	60
		2/15	40	21	968	49	688.00	3.86	692	0.58	14400.13	692	60
		1/6	41	22	996	50	674.27	4.16	677	0.30	14400.34	677	60
average					976.44	49.33	687.67	4.11	690.22	0.35	13262.24	690.22	60.00
226	5	1/10	34	18	824	42	999.64	4.20	1007	0.70	14400.06	1007	44
		2/15	35	19	852	43	852.00	3.67	853	0.12	6876.05	853	47
		1/6	36	19	872	44	879.32	4.28	882	0.23	14400.08	882	47
	7	1/10	34	17	816	41	915.07	4.03	917	0.11	3659.91	917	43
		2/15	37	19	892	45	859.91	5.84	865	0.58	14400.17	865	47
		1/6	35	18	844	43	843.06	2.94	845	0.12	3685.88	845	47
	10	1/10	34	17	816	41	837.19	3.09	840	0.24	14400.14	840	47
		2/15	37	19	892	45	859.91	5.88	865	0.58	14400.09	865	47
		1/6	35	18	844	43	843.06	2.94	845	0.12	3700.31	845	47
average					850.22	43.00	876.57	4.1	879.89	0.31	9991.41	879.89	46.22
238	5	1/10	49	26	1188	60	855.80	166.24	863	0.82	14400.33	863	60
		2/15	49	25	1180	59	812.27	15.02	821	0.98	14400.13	821	60
		1/6	48	25	1160	58	824.07	9.44	827	0.24	14400.28	827	60
	7	1/10	57	33	1404	71	739.61	97.20	754	1.89	14409.48	754	71
		2/15	54	30	1320	66	827.26	14.59	834	0.72	14400.45	834	70
		1/6	54	30	1320	66	694.23	11.14	698	0.43	14400.33	698	70
	10	1/10	56	32	1376	69	757.77	93.66	773	1.98	14400.16	773	70
		2/15	55	31	1348	68	813.43	83.79	832	2.21	14401.48	832	69
		1/6	54	30	1320	66	727.37	12.83	731	0.41	14400.19	731	70
average					1290.67	64.78	783.53	55.99	792.56	1.08	14401.43	792.56	66.67

Table 3: Computational results from DRP problems with $\lambda_1 = 0.96$ and $\lambda_2 = 0.04$

(1)	(2)	(3)	(4)			(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
instances	ε	*p*	nb crew duties			trivial lower	linear lower	CPU (sec)	optimal or upper	% gap	CPU (sec)	δ+ (min)	nb drivers
122	5	1/10	17	11	428	22	47.89	0.53	51.56	1.56	389.16	737	23
		2/15	17	11	428	22	49.55	0.50	53.00	1.09	315.72	725	25
		1/6	17	11	428	22	59.54	0.52	63.20	1.23	360.61	980	25
	7	1/10	17	11	428	22	54.82	0.48	58.32	1.07	305.97	858	25
		2/15	17	11	428	22	50.25	0.50	54.12	1.86	270.56	777	24
		1/6	17	10	420	21	41.59	0.89	45.92	2.00	532.33	572	24
	10	1/10	18	12	456	23	53.93	0.64	56.8	1.69	425.88	820	25
		2/15	18	11	448	23	53.06	0.58	55.56	1.05	329.47	789	25
		1/6	18	12	456	23	53.56	0.63	56.32	1.52	346.03	808	25
average					435.56	22.22	51.58	0.59	54.98	1.45	363.97	785.11	24.56
168	5	1/10	36	18	864	44	64.80	8.09	68.56	1.47	3452.52	658	44
		2/15	36	17	856	43	65.44	8.16	69.88	1.42	4497.92	715	43
		1/6	37	19	892	45	70.55	10.40	74.24	1.25	8891.95	752	46
	7	1/10	38	19	912	46	72.21	9.74	75.32	1.29	3612.17	779	46
		2/15	38	18	904	46	66.02	10.22	68.72	1.36	3621.59	590	47
		1/6	38	17	896	45	68.26	9.04	71.72	1.30	4944.44	689	46
	10	1/10	42	20	1000	50	65.98	3.73	70.00	1.41	6302.16	550	50
		2/15	42	20	1000	50	67.05	3.55	71.28	1.42	6458.44	582	50
		1/6	42	20	1000	50	67.27	3.73	71.52	1.39	10693.26	588	50
average					924.89	46.56	67.51	7.41	71.25	1.37	5830.49	655.89	46.89
224	5	1/10	41	18	964	49	74.31	13.42	80.56	1.47	14400.16	838	49
		2/15	41	20	980	49	80.02	4.31	86.76	1.64	14400.19	969	50
		1/6	41	19	972	49	72.57	12.41	78.40	1.38	14400.11	784	49
	7	1/10	40	23	984	50	74.07	13.65	79.44	1.27	8619.89	786	50
		2/15	39	21	948	48	75.90	11.61	83.56	1.94	14400.08	937	48
		1/6	41	21	988	50	72.58	12.44	77.72	1.27	7581.95	743	50
	10	1/10	41	21	988	50	75.68	14.39	81.36	1.23	8694.09	834	50
		2/15	40	21	968	49	74.56	12.83	81.04	8.01	14400.20	850	49
		1/6	41	22	996	50	74.97	12.58	80.52	1.22	10816.64	813	50
average					976.44	49.33	74.96	11.96	81.04	2.16	11968.15	839.33	49.44
226	5	1/10	34	18	824	42	80.31	13.16	83.04	1.07	5593.72	1044	43
		2/15	35	19	852	43	75.36	11.12	78.64	3.34	14400.11	934	43
		1/6	36	19	872	44	77.41	11.53	80.36	0.82	4386.48	953	44
	7	1/10	34	17	816	41	75.96	12.89	78.64	1.24	2378.44	958	42
		2/15	37	19	892	45	77.60	18.43	79.96	1.11	5972.00	895	46
		1/6	35	18	844	43	75.00	8.03	78.84	1.23	3591.36	915	44
	10	1/10	34	17	816	41	72.85	7.99	78.20	1.27	4934.52	971	41
		2/15	37	19	892	45	77.60	18.14	79.96	1.11	6002.75	895	46
		1/6	35	18	844	43	75.00	8.23	78.84	1.23	3638.98	915	44
average					850.22	43.00	76.34	12.17	79.61	1.38	5655.37	942.22	43.67
238	5	1/10	49	26	1188	60	91.83	18.77	92.84	1.09	1931.17	881	60
		2/15	49	25	1180	59	89.13	15.48	90.80	1.26	14400.38	854	59
		1/6	48	25	1160	58	88.64	9.28	90.40	0.74	5195.30	868	58
	7	1/10	57	33	1404	71	92.74	22.06	99.08	0.94	14393.80	773	71
		2/15	54	30	1320	66	96.45	13.97	100.40	2.07	14400.50	926	66
		1/6	54	30	1320	66	91.13	13.20	93.68	1.04	11681.90	758	66
	10	1/10	56	32	1376	69	96.55	80.22	97.96	0.99	10026.16	769	70
		2/15	55	31	1348	68	97.82	19.56	99.76	1.01	11186.97	862	68
		1/6	54	30	1320	66	92.45	14.72	94.72	0.64	9210.25	784	66
average					1290.67	64.78	92.97	23.03	95.52	1.09	10269.60	830.56	64.89

Table 4: Correlation coefficients with λ_1 =0.96 and λ_2 =0.04

instances	**correlation coefficients**
122	-0,0263
168	-0,0815
224	0,2089
226	0,0772
238	0,8491

Table 5: Correlation coefficients with $\lambda_1 = 1$ and $\lambda_2 = 0$

instances	**correlation coefficients**
122	0,6325
168	0,9456
224	-0,5595
226	0,9707
238	0,9940

Table 6: Correlation coefficients with $\lambda_1 = 0$ and $\lambda_2 = 1$

instances	**correlation coefficients**
122	-0,0473
168	-0,8067
224	0,2446
226	-0,3547
238	-0,5453